GRAPHICAL METHOD-SPECIAL CASES

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Graphical Method-Special cases

- Unique optimal solution
- Multiple optimal solution
- Unbound optimal solution
- No feasible solution

Basic Definition:

- **Feasible solution**
  - The set of values of decision variables Xj (j=1 2……n) which satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to be the feasible solution to that linear programming problem.
- Infeasible solution
  - The set of values of decision variables Xj (j=1 2……n) which do not satisfy all the constraints and non-negativity conditions of an LP problem simultaneously is said to constitute the infeasible solution to that linear programming problem.
- Basic solution
  - For a set of m simultaneous equations in n variables, a solution obtained by setting (n-m) variables equal to zero and solving for remaining m equations in m variables is called a basic solutions.

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- **Non basic and basic variable**
  - The (n-m) variables whose values did not appear in this solution are called non basic variable and the remaining m variables are called basic variables.
- **Basic feasible solution**
  - A feasible solution to an LP problem which is also the basic solution is called the basic feasible solution. All basic variables assume non-negative values.

Basic feasible solutions are of two types,

- (a) **Degenerate:** A basic feasible solution is called degenerate if value of at least one basic variable is zero.
- (b) **Non-degenerate:** A basic feasible solution is called non-degenerate if values all m basic variables are non-zero and positive.

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- **Optimum basic feasible solution**
  - A basic feasible solution which optimizes (Maximizes or Minimizes) the objective function value of the given LP problem is called an optimum basic feasible solution.
- **Unbounded solution**
  - A solution which increases or decreases the value of objective function of the LP problem indefinitely is called unbounded solution.

Procedure for solving LPP by Graphical Method:

- The steps involved in the graphical method are as follows.
- Step 1 Consider each inequality constraint as an equation.
- Step 2 Plot each equation on the graph as each will geometrically represent a straight line.
- Step 3 Mark the region. If the inequality constraint corresponding to that line is O then the region below the line lying in the first quadrant (due to non-negativity of variables) is shaded. For the inequality constraint P sign, the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. The common region, thus obtained, is called the feasible region.

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- Step 4 Assign an arbitrary value, say zero, for the objective function.
- Step 5 Draw a straight line to represent the objective function with the arbitrary value (i.e., a straight line through the origin).
- Step 6 Stretch the objective function line till the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin, passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passes through at least one corner of the feasible region.
- Step 7 Find the coordinates of the extreme points selected in step 6 and find the maximum or minimum value of Z.

Graphical Method-Special cases

1)Solve the following linear programming problem graphically:

$$\text{Maximise } Z = 4x + y \quad \dots (1)$$

subject to the constraints:

$$x + y \le 50 \quad \dots (2)$$

$$3x + y \le 90 \quad \dots (3)$$

$$x \ge 0,\ y \ge 0 \quad \dots (4)$$

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Solution:

- The shaded region in Fig 1 is the feasible region determined by the system of constraints (2) to (4).
- We observe that the feasible region OABC is bounded. So, we now use Corner Point Method to determine the maximum value of Z.
- The coordinates of the corner points O, A, B and C are (0, 0), (30, 0), (20, 30) and (0, 50) respectively.

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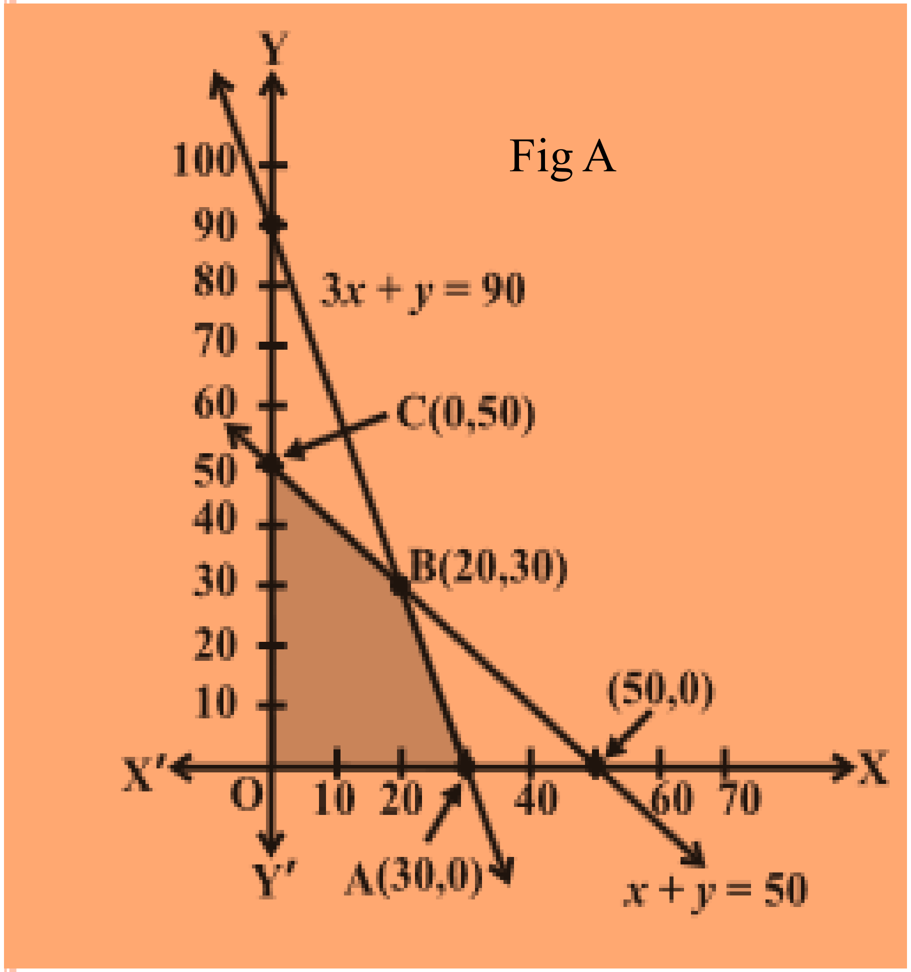
- Now we evaluate Z at each corner point.



Corner Point	Corresponding value of Z	
(0, 0)	0	
(30, 0)	**120** ←	Maximum
(20, 30)	110	
(0, 50)	50	

- Hence, maximum value of Z is 120 at the point (30, 0).

Case 1:UNIQUE OPTIMAL SOLUTION

1)Solve the following linear programming problem graphically:

Minimise $Z = 200x + 500y \ldots (1)$

subject to the constraints:

$x + 2y \geq 10 \ldots (2)$

$3x + 4y \leq 24 \ldots (3)$

and $x, y \geq 0 \ldots (4)$

Solution

- The shaded region in Fig B is the feasible region ABC determined by the system of constraints (2) to (4), which is bounded.
- The coordinates of corner points A, B and C are (0,5), (4,3) and (0,6) respectively.
- Now we evaluate Z = 200x + 500y at these points.

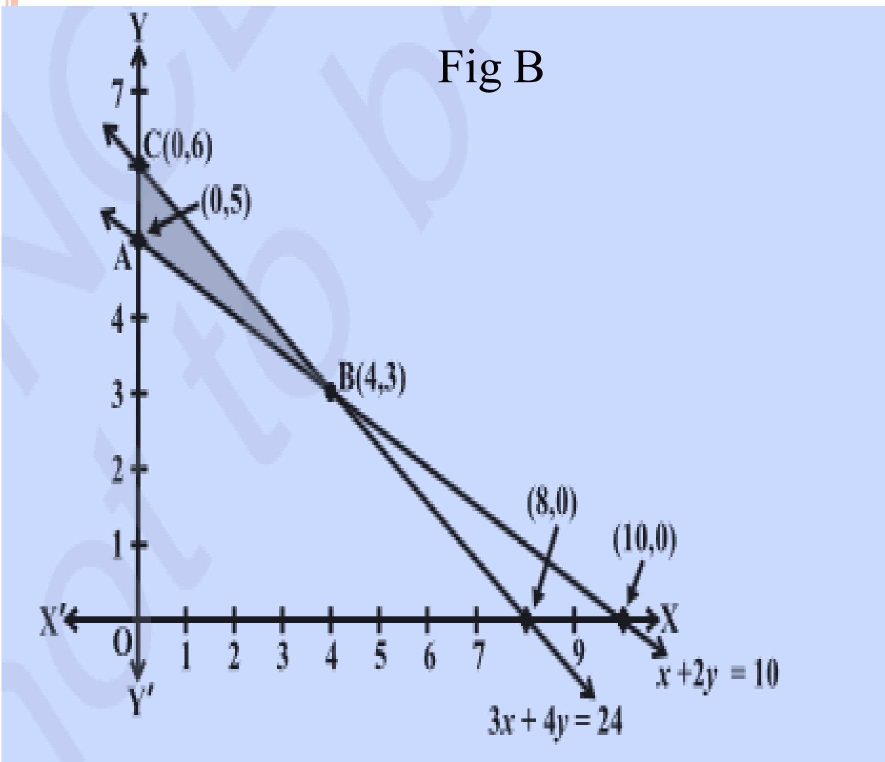


Fig B

Corner Point	Corresponding value of Z	
(0, 5)	2500	
(4, 3)	**2300** ←	Minimum
(0, 6)	3000	

Hence, minimum value of Z is 2300 attained at the unique point (4, 3) .

Case2:MULTIPLE OPTIMAL SOLUTION

3) Solve by using graphical method

Max $Z = 4x_1 + 3x_2$

Subject to $4x_1 + 3x_2 \leq 24$

$x_1 \leq 4.5$

$x_2 \leq 6$

and $x_1 , x_2 \geq 0$

Cont……

Solution:

- The first constraint $4x_1 + 3x_2 \leq 24$, written in a form of equation $4x_1 + 3x_2 = 24$

  Put $x_1 = 0$, then $x_2 = 8$

  Put $x_2 = 0$, then $x_1 = 6$
- The coordinates are (0, 8) and (6, 0)
- The second constraint $x_1 \leq 4.5$, written in a form of equation $x_1 = 4.5$
- The third constraint $x_2 \leq 6$, written in a form of equation $x_2 = 6$

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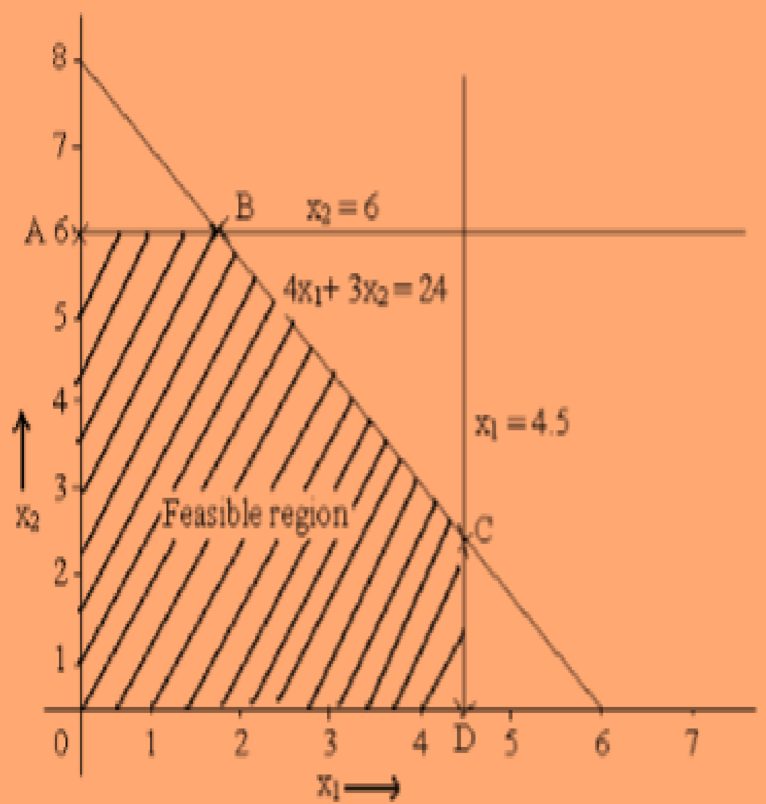

Corner points	Corresponding value of $Z = 4x_1 + 3x_2$
A (0, 6)	Z = 4(0) + 3(6) = 18
B (1.5, 6)- (Solve the two equations 4x1+ 3x2 = 24 and x2 = 6 to get the coordinates)	Z = 4(1.5) + 3(6) = 24
C (4.5, 2)- (Solve the two equations 4x1+ 3x2 = 24 and x1 = 4.5 to get the coordinates)	Z = 4(4.5) + 3(2) = 24
D (4.5, 0)	Z = 4(4.5) + 3(0) = 18

- The corner points of feasible region are A, B, C and D.
- Max Z = 24, which is achieved at both B and C corner points. It can be achieved not only at B and C but every point between B and C.
- Hence the given problem has multiple optimal solutions.

Case 3:UNBOUNDED SOLUTION

4)Solve by graphical method

$\text{Max } Z = 3x_1 + 5x_2$

Subject to $2x_1 + x_2 \geq 7$

$x_1 + x_2 \geq 6$

$x_1 + 3x_2 \geq 9$

and $x_1 , x_2 \geq 0$

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Solution:

- The first constraint $2x_1 + x_2 \geq 7$, written in a form of equation $2x_1 + x_2 = 7$

  Put $x_1 = 0$, then $x_2 = 7$

  Put $x_2 = 0$, then $x_1 = 3.5$

  The coordinates are (0, 7) and (3.5, 0)

- The second constraint $x_1 + x_2 \geq 6$, written in a form of equation $x_1 + x_2 = 6$

  Put $x_1 = 0$, then $x_2 = 6$

  Put $x_2 = 0$, then $x_1 = 6$

  The coordinates are (0, 6) and (6, 0)

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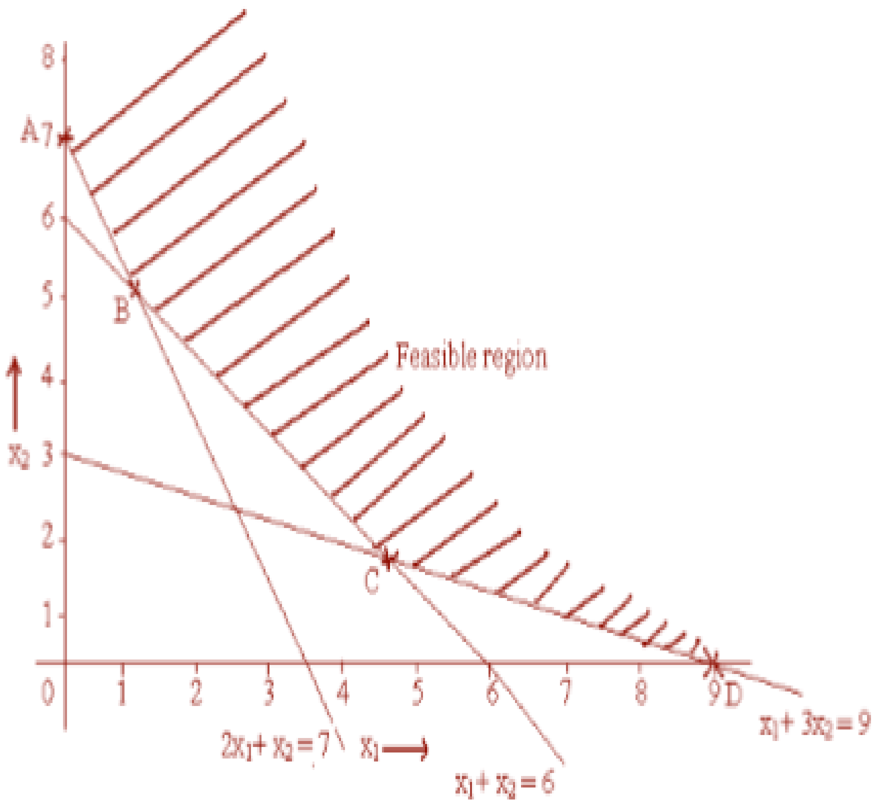
- The third constraint x1+ 3x2 ≥ 9, written in a form of equation $x_1 + 3x_2 = 9$

  Put $x_1 = 0$, then $x_2 = 3$

  Put $x_2 = 0$, then $x_1 = 9$ The coordinates are (0, 3) and (9, 0)



Corner points	Corresponding value of $Z = 3x_1 + 5x_2$
A (0, 7)	Z = 3(0) + 5(7) = 35
B (1, 5) (Solve the two equations $2x_1 + x_2 = 7$ and $x_1 + x_2 = 6$ to get the coordinates)	Z = 3(1) + 5(5) = 28
C (4.5, 1.5) (Solve the two equations $x_1 + x_2 = 6$ and $x_1 + 3x_2 = 9$ to get the coordinates)	Z = 3(4.5) + 5(1.5) = 21
D (9, 0)	Z = 3(9) + 5(0) = 27

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- The corner points of feasible region are A, B, C and D.
- The values of objective function at corner points are 35, 28, 21 and 27.
- But there exists infinite number of points in the feasible region which is unbounded.
- The value of objective function will be more than the value of these four corner points i.e. the maximum value of the objective function occurs at a point at ∞.
- Hence the given problem has unbounded solution.

Case 4:NO FEASIBLE SOLUTION

5)Solve graphically

$\text{Max } Z = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 1$

$x_1 + x_2 \geq 3$

and $x_1 , x_2 \geq 0$

Cont……

Solution

- The first constraint $x_1 + x_2 \leq 1$, written in a form of equation $x_1 + x_2 = 1$

  Put $x_1 = 0$, then $x_2 = 1$

  Put $x_2 = 0$, then $x_1 = 1$

  The coordinates are (0, 1) and (1, 0)

- The first constraint $x1 + x2 \geq 3$, written in a form of equation $x_1 + x_2 = 3$

  Put $x_1 = 0$, then $x_2 = 3$

  Put $x_2 = 0$, then $x_1 = 3$

  The coordinates are (0, 3) and (3, 0)

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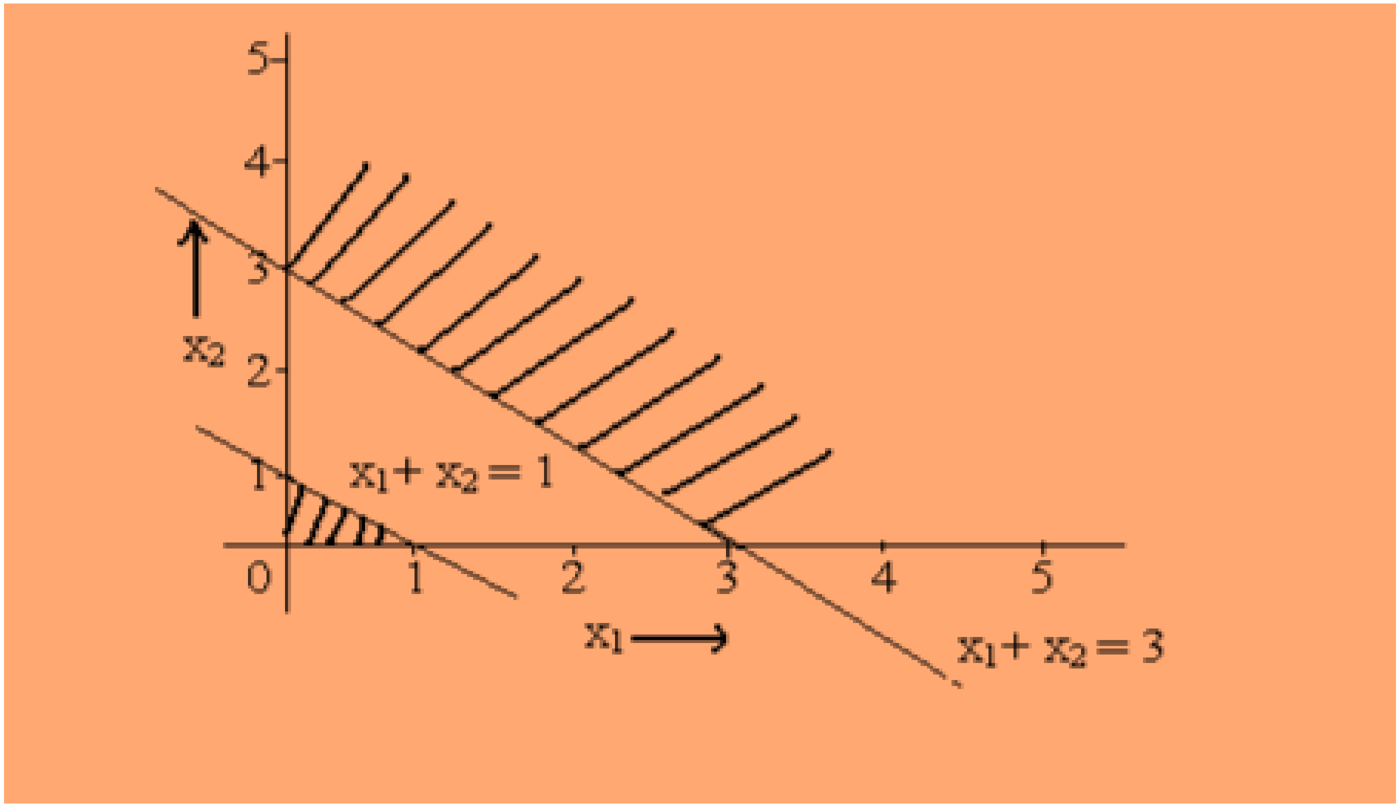

- There is no common feasible region generated by two constraints together i.e. we cannot identify even a single point satisfying the constraints.
- Hence there is no feasible solution.

Thank You